$\frac{30}{x-7}$=time up stream with barge; $\frac{30}{x+1}$=time in returning down stream alone.

$$\frac{30}{x-7}+\frac{30}{x+1}=12\tfrac{8}{11}=\tfrac{140}{11},$$

or, clearing of fractions, $330x+330+330x-2310=140x^2-840x-980$.

Whence, $7x^2-75x+50=0$, and therefore $7x=70$ or 5.

$x=10$ or $\frac{5}{7}$. $\frac{5}{7}$ is not admissible.

Also solved by M. A. Muzzy, A. H. Holmes, J. Scheffer, and H. C. Feemster.

366. Proposed by WILLIAM HOOVER, Ph. D., Professor of Mathematics and Astronomy, Ohio University Athens, Ohio.

Eliminate m from the equations

$$3a^2m^4+4aym^3+6axm^2-(x^2+y^2-4ax)=0;$$
$$(x^2+y^2-4ax)m^4-6axm^2+4aym-3a^2=0.$$

Solution by J. SCHEFFER, A. M., Hagerstown, Maryland.

Dividing the first equation by m^4, and adding, we get

$$4ay(m+\frac{1}{m})-6ax(m^2-\frac{1}{m^2})+(x^2+y^2-4ax)\ (m^4-\frac{1}{m^4})$$

and dividing this by $m+\frac{1}{m}$, we have

$$4ay-6ax(m-\frac{1}{m})+(x^2+y^2-4ax)\ (m^2+\frac{1}{m^2})\ (m-\frac{1}{m})=0.$$

Putting $m-\frac{1}{m}=p$, the last expression reduces to

$$4ay-6axp+(x^2+y^2-4ax)(p^2+2)p=0\ldots(\mathrm{I}).$$

Again dividing both of the original equations by m^2 and adding, we have

$$3a^2(m^2-\frac{1}{m^2})+4ay(m+\frac{1}{m})+(x^2+y^2-4ax)\ (m^2-\frac{1}{m^2})=0.$$

Suppressing the factor $m+\frac{1}{m}$, we get

$$3a^2\left(m-\frac{1}{m}\right)+4ay+(x^2+y^2-4ax)\left(m-\frac{1}{m}\right)=0; \text{ or}$$

$$3a^2p+4ay+(x^2+y^2-4ax)p=0\ldots(\text{II}).$$

From (II), we get $p=-\dfrac{4ay}{x^2+y^2-4ax+3a^2}$, and substituting this in (I), we get

$$(x^2+y^2+3a^2-4ax)^2(3a^2+10ax-x^2-y^2)-16a^2y^2(x^2+y^2-4ax)=0.$$

Also solved by M. A. Harding, S. Lefschetz, and A. H. Holmes.

367. Proposed by E. B. ESCOTT, University of Michigan, Ann Arbor, Michigan.

Solve the simultaneous equations:

$$\frac{2x}{1+x^2}=y\ldots(1);\quad \frac{2y}{1+y^2}=z\ldots(2);\quad \frac{2z}{1+z^2}=u\ldots(3);\quad \frac{2u}{1+u^2}=x\ldots(4).$$

Solution by PROFESSOR F. L. GRIFFIN, Reed College, Portland, Oregon.

By inspection three solutions are $x=y=z=u=1$, or -1, or 0; and there can be but fourteen others. Now let $x=i\tan\theta$, $[i=\sqrt{-1}]$, whence $y=i\tan 2\theta$, $z=i\tan 4\theta$, $u=i\tan 8\theta$, and $x=i\tan 16\theta$. But $\tan 16\theta=\tan\theta$ for finite values of θ only if $16\theta=\theta+n\pi$, or $\theta=n\pi/15$. Thus we have

$$\begin{array}{llllll}
x=0, & i\tan\pi/15, & i\tan 2\pi/15, & i\tan 3\pi/15, & \ldots, & i\tan 14\pi/15;\\
y=0, & i\tan 2\pi/15, & i\tan 4\pi/15, & i\tan 6\pi/15, & \ldots, & i\tan 28\pi/15;\\
z=0, & i\tan 4\pi/15, & i\tan 8\pi/15, & i\tan 12\pi/15, & \ldots, & i\tan 56\pi/15;\\
u=0, & i\tan 8\pi/15, & i\tan 16\pi/15, & . \quad . \quad . & . \quad . & . \quad .
\end{array}$$

the values for y, z, u being of course the same sets as for x in different orders. The values $x=+1$, -1 correspond to infinite values of θ for which $\tan\theta=-i$, $+i$.

GEOMETRY.

393. Proposed by S. LEFSEHETZ, University of Nebraska.

Draw a triangle having a given angle, and with its vertices on three given concentric circles.

I. Solution by H. C. FEEMSTER, A. B., Professor of Mathematics, York College, York, Nebraska.

Let $x^2+y^2=c^2$, $x^2+y^2=b^2$, $x^2+y^2=a^2$, be the given circles, and ϕ the given angle. Place the vertex of the given angle, ϕ, on the circumference of the first circle at 0, c, the angle being formed by the lines,